Continuous-state Branching Processes: Genealogy, Duality and Interaction: contents

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Continuous-state branching processes (CSBPs) have been defined in the sixties. They form the continuous-state continuous-time analogue of Galton-Watson processes. They are cornerstone in the understanding of stochastic population models and appear also in many different stochastic models. The purpose of this short course is to shed light on two of their aspects: the backwards genealogy of CSBPs, and the boundary behaviors of CSBPs with quadratic competition. Both studies are relying on the concept of stochastic duality (which we shall specify later). The results are extirped from the following articles:
For the two first sessions:

- Coalescences in Continuous-state branching processes with Chunhua Ma and Bastien Mallein (EJP 2019),
- Asymptotic behaviour of ancestral lineages in subcritical continuous-state branching populations, with Martin Möhle (SPA 2022)

The last session is based on the works:

- Continuous-state branching processes with competition: duality and reflection at infinity (EJP 2019) and
- Local explosions and extinction in continuous-state branching processes with logistic competition (preprint available on ArXiv).

More details on the contents are below.

Talk 1: CSBPs, Ancestral Lineages and Siegmund duality

In the first session, I will recall briefly some well-known facts on CSBPs, as for instance: the representation of their semigroup, of the generator and the classification of the boundaries. I will refer to Zenghu Li's lecture notes and Andreas Kyprianou's book for those results and do not provide proofs. I shall introduce some of the basic notations and explain heuristically what we are going to study. Our starting point will be a construction due to Bertoin and Le Gall of a complete population model (i.e. where individuals are specified) for CSBPs (existence will be omitted, I refer to Bertoin's lecture notes). We shall end the session by defining a new process that represents the ancestral lineage of individuals backwards in time. This corresponds to the inverse flow of the CSBPs, we call them Ancestral Lineage Processes (ALPs), and provide some first information on their semigroup, longterm behaviors and generators. Depending on the time left, I shall give details on the proof leading to the generator form of ALPs.

Talk 2: Genealogy and Coalescent in CSBPs

The objective of this session is to explain how ancestral lineages merge as time goes backwards. We shall see how to sample individuals in the current generation along an independent Poisson process, and how lineages of those individuals merge. The coalescent theory induced behind is rather elementary and shares many features with flows of bridges and exchangeable coalescents. I plan to provide some details on how the coalescent process evolves in time. The long term behavior of the partition-valued coalescent will be discussed; in the subcritical case, we shall see that it converges almost surely towards a partition with infinitely many blocks each independent, whose size law is related to the quasi-stationary distribution. Last, we will state some results on the long-term behavior of the ALP processes in the subcritical case and find an almost sure renormalisation of it.

Talk 3: Logistic CSBPs, Laplace duality and reflection at infinity.

In the last session, we change of topic and introduce a new force in the CSBPs; the so-called logistic competition (or quadratic competition). It is modelling a phenomenon of pairwise fight between individuals in the population (in terms of population modelling, this can be viewed as modelling limited ressources). The process with competition (LCSBP) does not satisfy the branching property; but we shall be able to study it through another duality relationship with a certain one-dimensional diffusion on $(0, \infty)$: the so-called Laplace duality. We shall classify the behaviors of the process at its boundary infinity with the help of the dual. A phenomenon of instantaneous reflection at infinity is revealed. The construction we give of LCSBP reflected at infinity is made by taking limits of processes with boundary entrance; I will spend some time on this construction. The latter is not providing any information on the excursion measure, I shall state some results on the local time at infinity of the LCSBP by introducing the Siegmund dual process of the Laplace dual.